ISE 220 – Engineering Economics

Time Value Of Money - 3

University of Economics

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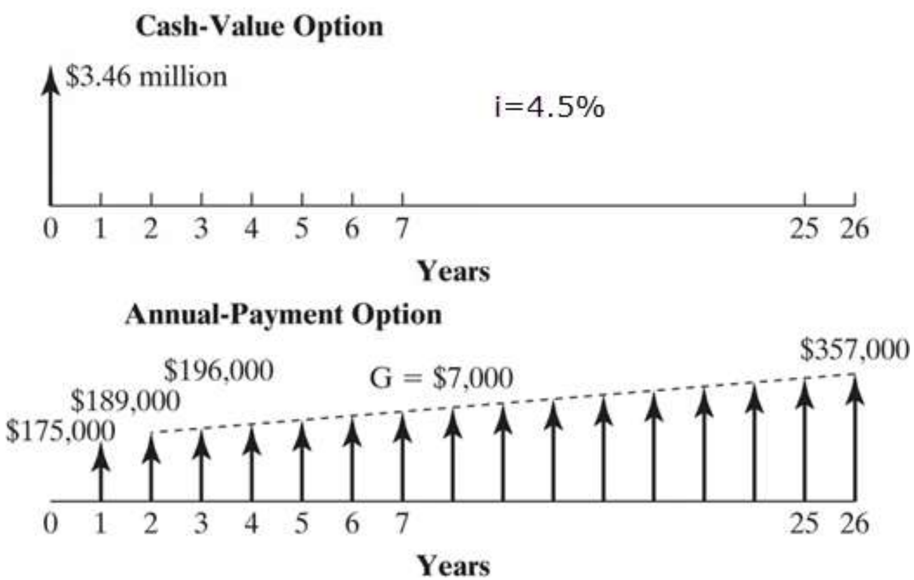
Agenda

- Economic Equivalence
- Single Cash Flow
- Uneven payment series
- Equal Payment Series
- Linear Gradient Series
- Geometric Gradient Series
- Composite Cash Flows
- Quiz

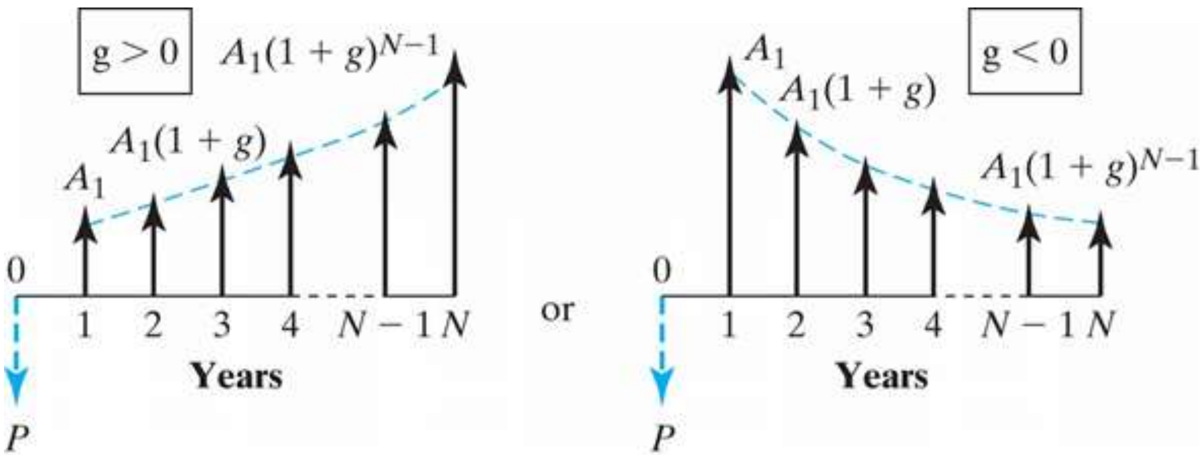
Solution

Example 2.18



	A	B	C
1	Example 2.18		
2			
3	*i*=	4.50%	0.045
4	N=	26	Years
5	A_1=	175000	$
6	A_2=	189000	$
7	G=	7000.00	$
8			
9	P=	=-NPV(B3;B13:B38)	
10			
11	Years	Cash Flow	Cash Balance
12	0	-$3,818,363.12	-$3,818,363.12
13	1	$175,000.00	-$3,815,189.46
14	2	$189,000.00	-$3,797,872.98
15	3	$196,000.00	-$3,772,777.27
16	4	$203,000.00	-$3,739,552.24
17	5	$210,000.00	-$3,697,832.09
18	6	$217,000.00	-$3,647,234.54
19	7	$224,000.00	-$3,587,360.09
20	8	$231,000.00	-$3,517,791.30
21	9	$238,000.00	-$3,438,091.91
22	10	$245,000.00	-$3,347,806.04
23	11	$252,000.00	-$3,246,457.31
24	12	$259,000.00	-$3,133,547.89
25	13	$266,000.00	-$3,008,557.55
26	14	$273,000.00	-$2,870,942.64
27	15	$280,000.00	-$2,720,135.06
28	16	$287,000.00	-$2,555,541.13
29	17	$294,000.00	-$2,376,540.48
30	18	$301,000.00	-$2,182,484.81
31	19	$308,000.00	-$1,972,696.62
32	20	$315,000.00	-$1,746,467.97
33	21	$322,000.00	-$1,503,059.03
34	22	$329,000.00	-$1,241,696.68
35	23	$336,000.00	-$961,573.04
36	24	$343,000.00	-$661,843.82
37	25	$350,000.00	-$341,626.79
38	26	$357,000.00	$0.00

Geometric Gradient Series



Increasing Geometric Series **Decreasing Geometric Series**

Geometric gradient Present worth $(P/A_1, g, i, N)$

$$P = \begin{bmatrix} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \\ A_1\left(\dfrac{N}{1+i}\right), (\text{if } i = g) \end{bmatrix}$$

Geometric Gradient Series

$P = A_1(P/A, g, i, N)$ →

	Single Payment		Equal Payment Series				Gradient Series		
N	Compound Amount Factor (*F/P,i,N*)	Present Worth Factor (*P/F,i,N*)	Compound Amount Factor (*F/A,i,N*)	Sinking Fund Factor (*A/F,i,N*)	Present Worth Factor (*P/A,i,N*)	Capital Recovery Factor (*A/P,i,N*)	Gradient Uniform Series (*A/G,i,N*)	Gradient Present Worth (*P/G,i,N*)	*N*
1	1.1300	0.8850	1.0000	1.0000	0.8850	1.1300	0.0000	0.0000	1
2	1.2769	0.7831	2.1300	0.4695	1.6681	0.5995	0.4695	0.7831	2
3	1.4429	0.6931	3.4069	0.2935	2.3612	0.4235	0.9187	2.1692	3
4	1.6305	0.6133	4.8498	0.2062	2.9745	0.3362	1.3479	4.0092	4
5	1.8424	0.5428	6.4803	0.1543	3.5172	0.2843	1.7571	6.1802	5
6	2.0820	0.4803	8.3227	0.1202	3.9975	0.2502	2.1468	8.5818	6

Geometric gradient Present worth $(P/A_1, g, i, N)$

$$P = \begin{bmatrix} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \\ A_1\left(\dfrac{N}{1+i}\right), (\text{if } i = g) \end{bmatrix}$$

$g' = (i - g) / (1 + g)$

$P = [A_1/(1+g)](P/A, g', N)$

Geometric Gradient Series

Example 2.19

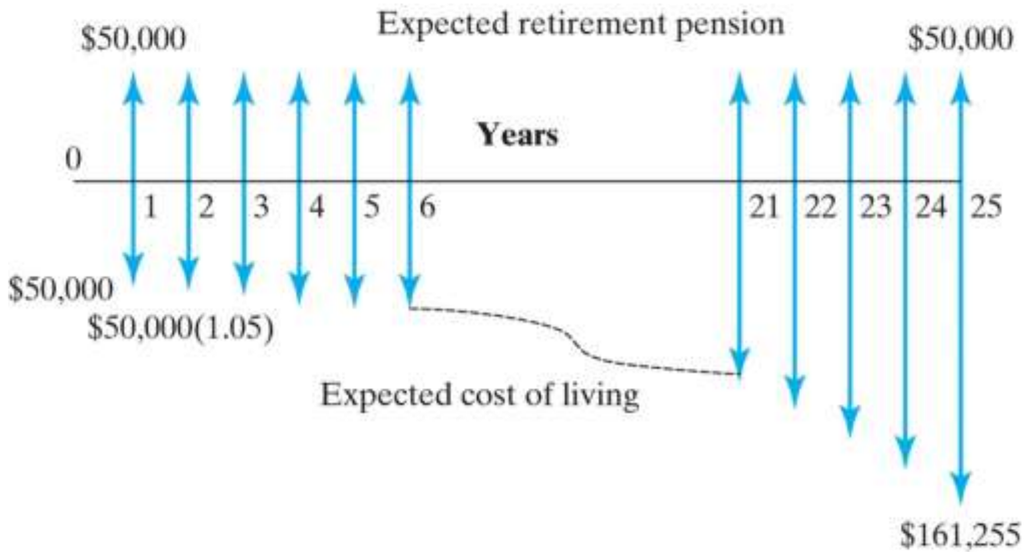
EXAMPLE 2.19 Required Cost-of-Living Adjustment Calculation

Suppose that your retirement benefits during your first year of retirement are \$50,000. Assume that this amount is just enough to meet your cost of living during the first year. However, your cost of living is expected to increase at an annual rate of 5% due to inflation. Suppose you do not expect to receive any cost-of-living adjustment in your retirement pension. Then, some of your future cost of living has to come from savings other than retirement pension. If your savings account earns 7% interest a year, how much should you set aside in order to meet this future increase in the cost of living over 25 years?

DISSECTING THE PROBLEM	**Given:** $A_1 = \$50{,}000, g = 5\%, i = 7\%$ per year, and $N = 25$ years, as shown in Figure 2.31. **Find:** P.

Geometric Gradient Series

Example 2.19



g = 5%
i = 7%

$$P = \$50{,}000(P/A_1, 5\%, 7\%, 25)$$
$$= \$50{,}000\left[\frac{1 - (1+0.05)^{25}(1+0.07)^{-25}}{0.07 - 0.05}\right]$$
$$= \$50{,}000(18.8033)$$
$$= \$940{,}167.22.$$

Geometric gradient Present worth $(P/A_1, g, i, N)$

$$P = \begin{bmatrix} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \\ A_1\left(\dfrac{N}{1+i}\right), (\text{if } i = g) \end{bmatrix}$$

P_1 = 50000(P/A, 7%, 25) = 582679

g’ = (0.07 – 0.05) / (1 + 0.05) = 1.9048%
P = [50000 / (1 + 0.05)] (P/A, 1.9048%, 25)
P_2 = 940167

Delta P = **357488**

...nt Series		Gradient Series		
Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	N
0.9804	1.0200	0.0000	0.0000	1
1.9416	0.5150	0.4950	0.9612	2
2.8839	0.3468	0.9868	2.8458	3
3.8077	0.2626	1.4752	5.6173	4

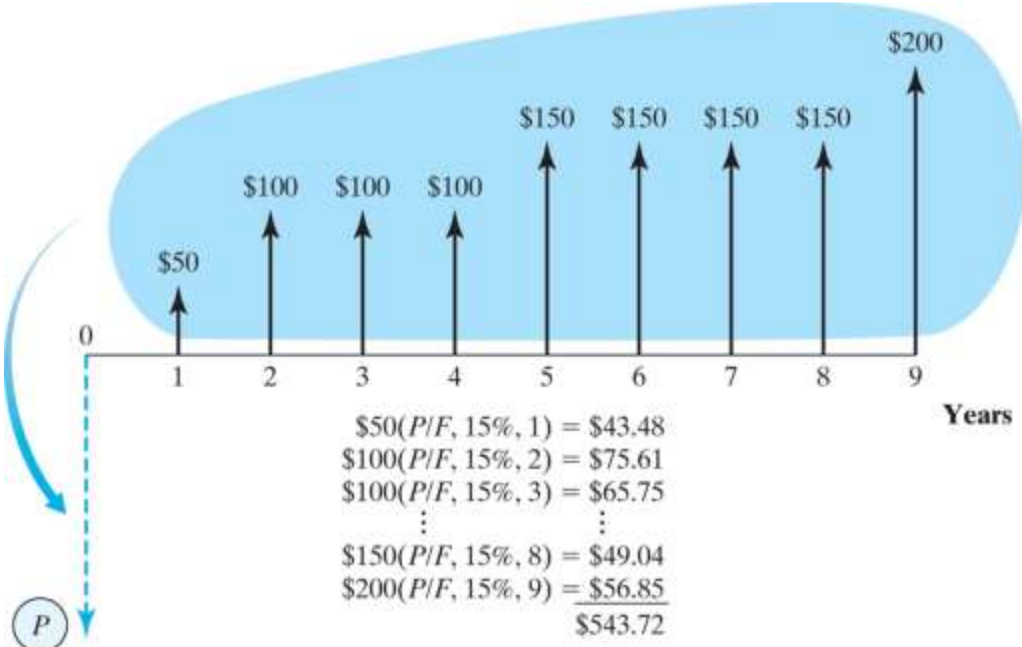
2.0%

Geometric Gradient Series

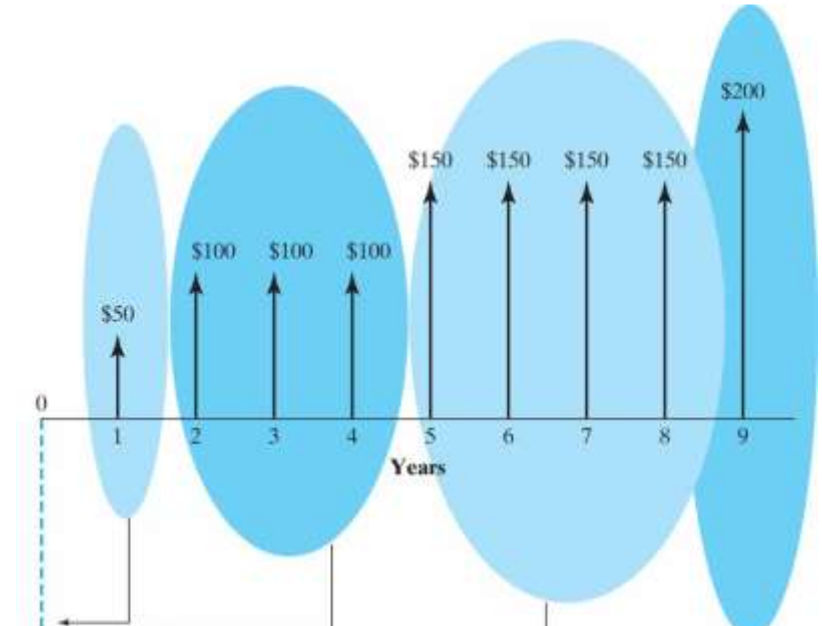
Example 2.19

	A	B	C	D	E	F
1	Example 2.18					
2	i=	7.00%	0.07			
3	g=	5.00%	0.05			
4	N=	25	Years			
5	A=	50000				
6	P1=	-$582,679.16				
7	P2=	=-NPV(B2;E11:E35)				
8						
9	Years	Cash Flow 1	Cash Balance 1		Cash Flow 2	Cash Balance 2
10	0	-$582,679.16	-$582,679.16		-$940,167.22	-$940,167.22
11	1	$50,000.00	-$573,466.70		$50,000.00	-$955,978.92
12	2	$50,000.00	-$563,609.37		$52,500.00	-$970,397.45
13	3	$50,000.00	-$553,062.02		$55,125.00	-$983,200.27
14	4	$50,000.00	-$541,776.37		$57,881.25	-$994,143.04
15	5	$50,000.00	-$529,700.71		$60,775.31	-$1,002,957.74
16	6	$50,000.00	-$516,779.76		$63,814.08	-$1,009,350.70
17	7	$50,000.00	-$502,954.35		$67,004.78	-$1,013,000.47
18	8	$50,000.00	-$488,161.15		$70,355.02	-$1,013,555.48
19	9	$50,000.00	-$472,332.43		$73,872.77	-$1,010,631.59
20	10	$50,000.00	-$455,395.70		$77,566.41	-$1,003,809.39
21	11	$50,000.00	-$437,273.40		$81,444.73	-$992,631.32
22	12	$50,000.00	-$417,882.54		$85,516.97	-$976,598.54
23	13	$50,000.00	-$397,134.31		$89,792.82	-$955,167.63
24	14	$50,000.00	-$374,933.72		$94,282.46	-$927,746.90
25	15	$50,000.00	-$351,179.08		$98,996.58	-$893,692.61
26	16	$50,000.00	-$325,761.61		$103,946.41	-$852,304.68
27	17	$50,000.00	-$298,564.93		$109,143.73	-$802,822.28
28	18	$50,000.00	-$269,464.47		$114,600.92	-$744,418.92
29	19	$50,000.00	-$238,326.98		$120,330.96	-$676,197.28
30	20	$50,000.00	-$205,009.87		$126,347.51	-$597,183.58
31	21	$50,000.00	-$169,360.56		$132,664.89	-$506,321.55
32	22	$50,000.00	-$131,215.80		$139,298.13	-$402,465.93
33	23	$50,000.00	-$90,400.91		$146,263.04	-$284,375.51
34	24	$50,000.00	-$46,728.97		$153,576.19	-$150,705.60
35	25	$50,000.00	-$0.00		$161,255.00	-$0.00

Composite Cash Flows



	A	B	C
1	Composite Cash Flow		
2	i=	0.15	15.00%
3	N=	9	Years
4			
5	P	543.72TL	TL
6			
7	Period (n)	Cash Flow	PV
8	0		0
9	1	50.00TL	43.48TL
10	2	100.00TL	75.61TL
11	3	100.00TL	65.75TL
12	4	100.00TL	57.18TL
13	5	150.00TL	74.58TL
14	6	150.00TL	64.85TL
15	7	150.00TL	56.39TL
16	8	150.00TL	49.04TL
17	9	200.00TL	56.85TL



$P_{Group1} = \$50(P/F, 15\%, 1)$
$= \$43.48$

$P_{Group2} = \$100(P/A, 15\%, 3)(P/F, 15\%, 1)$
$= \$198.54$

$P_{Group3} = \$150(P/A, 15\%, 4)(P/F, 15\%, 4)$
$= \$244.85$

$P_{Group4} = \$200(P/F, 15\%, 9)$
$= \$56.85$

$P = \$43.48 + \$198.54 + \$244.85 + \$56.85$
$= \$543.72$

Composite Cash Flows

EXAMPLE 2.20 Retirement Planning: Composite Series That Requires Multiple Interest Factors

You want to supplement your retirement income through IRA contributions. You have 15 years left until retirement and you are going to make 15 equal annual deposits into your IRA until you retire with the first deposit being made at the end of year 1. You need to save enough so that you can make 10 annual withdrawals that will begin at the end of year 16. The first withdrawal will be \$10,000, and each subsequent withdrawal will increase at a rate of 4% over the previous year's withdrawal in line with expected increase in cost-of-living. Your last withdrawal will be at the end of year 25. What is the amount of the equal annual deposit amount (C) for the first 15 years? Assume the interest rate is 8% compounded annually before and after you retire.

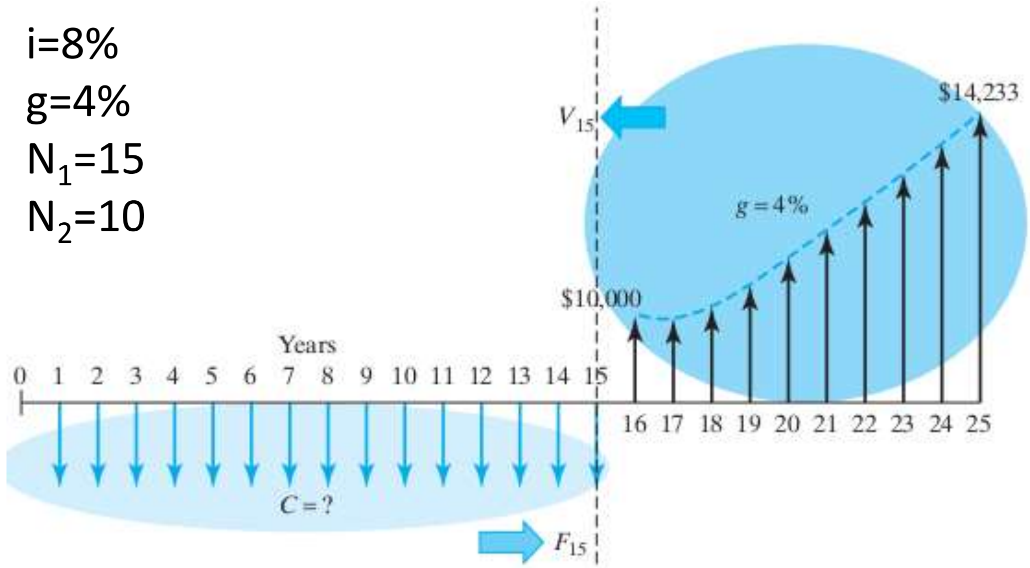
DISSECTING THE PROBLEM

Given: $i = 8\%$ per year, deposit and withdrawal series shown in Figure 2.34a.
Find: A

Composite Cash Flows

2.20

i=8%
g=4%
N_1=15
N_2=10



$g' = (i - g) / (1 + g)$

$P = [A_1/(1+g)](P/A, g', N)$

Geometric gradient Present worth $(P/A_1, g, i, N)$

$$P = \begin{bmatrix} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \\ A_1\left(\dfrac{N}{1+i}\right), (\text{if } i = g) \end{bmatrix}$$

Quiz

Questions

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